Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Problem Solving-II
Subject Code: 5SC03PRS1 **Branch: M.Sc.(Mathematics)**
Semester: 3 **Date : 20/03/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Generate a field of order 27. **(02)**

b. Let $\beta = (1\ 3\ 5\ 7\ 9\ 8\ 6)(2\ 4\ 10)$. What is the smallest positive integer n for which $\beta^n = \beta^{-5}$? **(02)**

c. Suppose $\phi: z_{30} \to z_{30}$ is homomorphism and $\ker \phi = \{0, 10, 20\}$. If $\phi(13) = 9$ then determine all elements that map to 9. **(02)**

d. True or False : Any infinite cyclic group is isomorphic to Z_n. **(01)**

Q-2 **Attempt all questions** **(14)**

a) Determine the number of elements of order 5 in $z_{25} \oplus z_5$. **(06)**

b) Find all maximal ideal in $\boldsymbol{Z}_{10}$. **(04)**

c) Define: Conjugate Class. Also find the conjugate classes and class equation of S_3. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a) i. Check that $f(x) = x^4 + x^2 + 1$ is reducible over Z_2 ? **(06)**

ii. Compute $5^{15}\ mod\ 7$ and $7^{13}\ mod\ 11$. Also state result which you use.

b) How many homomorphism are there from z_{12} to z_{30} ? List all homomorphism. **(04)**

c) Find all units of $J[i]$. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Solve the given equation by Gauss-Elimination method **(06)**

$$2x - y + 2z = 2,\ x + 10y - 3z = 5,\ x - y - z = 3.$$

b) Given $y_0 = 1, y_1 = 1.5, y_2 = 2.2, y_3 = 3.1, y_4 = 4.6$. Evaluate $\Delta^3 y_1$ and y_5 by forming a forward difference table. **(04)**

c) If $O(G) = 15$, then how many 3-Sylow subgroup and 5-Sylow subgroups in G. Which of them are normal? **(04)**

OR

Q-3 **Attempt all questions** **(14)**

a) Find missing term in the following table. **(06)**

x	1	2	3	4	5	6	7
y	2	4	8	-	32	64	128

b) Let $G = \{\begin{bmatrix} a & a \\ a & a \end{bmatrix} : a \in \boldsymbol{R}, a \neq 0\}$ show that G is a group under multiplication. **(04)**

c) i) If a is an element of group G and $|a| = 7$, show that a is a cube of some element of G. **(04)**

ii) Let $H = \left\{\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} : a, b, d \in R, ad \neq 0\right\}$. Is H normal subgroup of $GL(2: R)$?

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Define : Boolean Ring and give an example of it. **(02)**

b. Find particular integral for $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$ **(02)**

c. Classify the following partial differential equations: **(02)**

i. $3u_{xx} + 4u_{xy} + 2u_{yy} = 5$

ii. $4u_{xx} + 16u_{xy} + 16u_{yy} = 0$

d. Evaluate :$\Delta \log(f(x))$. **(01)**

Q-5 **Attempt all questions** **(14)**

a) Find the integral surface of the partial differential equation **(06)**
$(x - y)p + (y - x - z)q = z$, passing through the circle $z = 1, x^2 + y^2 = 1$.

b) Solve:$xp + yq = pq$ **(04)**

c) Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 3x$ **(04)**

OR

Q-5 **Attempt all questions** **(14)**

a) Apply Stirling's formula to find a polynomial of degree four which takes **(07)**

x	1	2	3	4	5
$f(x)$	1	-1	1	-1	1

b) Solve : $z^2(p^2 + q^2 + 1) = 1$. **(04)**

c) Find the characteristics of $4u_{xx} + 45 + 3u_{yy} + u_x + u_y = 2$ **(03)**

Q-6 **Attempt all questions** **(14)**

a) Find the solution of $\frac{dy}{dx} = e^x - y$ up to the fifth approximation. Using Picard's method given that $y(0) = 0$. **(05)**

b) Solve: $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$. **(05)**

c) Consider the initial value problem $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, u(0, y) = 4e^{-2y}$ then find $u(x, y)$ and $u(1,1)$. **(04)**

OR

Q-6 **Attempt all Questions**

a) Determine $y(0.1)$ and $y(0.2)$ correct to four decimal places from **(07)**
$\frac{dy}{dx} = 2x + y, y(0) = 1$. Use fourth order Runge-Kutta method.

b) By using method of separation of variable solve two dimensional wave equation. **(07)**